Week 9 5PM
Tuesday, May 26, 2020 5:01 PM

*Course Evaluations

We can change from Cartesian to polar coordinates using

$$\boxed{x = r\cos(\theta) \qquad y = r\sin(\theta)}$$

$u = f(x)$

$du = f'(x)\,dx$

Jacobian

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix}$$

Change of Variables Formula

$$\to \iint_D f(x,y)\,dA = \iint_{D^*} f(x(u,v), y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dA^*$$

Ex Evaluate $\iint_D e^{\boxed{x^2+y^2}}\,dA$ where $D = \{(x,y) : 1 \le \boxed{x^2+y^2} \le 2,\ y \ge 0\}$

$x = r\cos\theta \qquad y = r\sin\theta$

$$e^{r^2\cos^2\theta + r^2\sin^2\theta} = e^{r^2(\cos^2\theta + \sin^2\theta)} = e^{r^2}$$

$\boxed{\cos^2\theta + \sin^2\theta = 1}$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \partial x/\partial r & \partial x/\partial \theta \\ \partial y/\partial r & \partial y/\partial \theta \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta$$

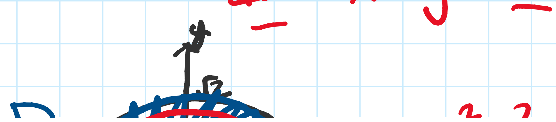
$= r(\cos^2\theta + \sin^2\theta)$

$= r$

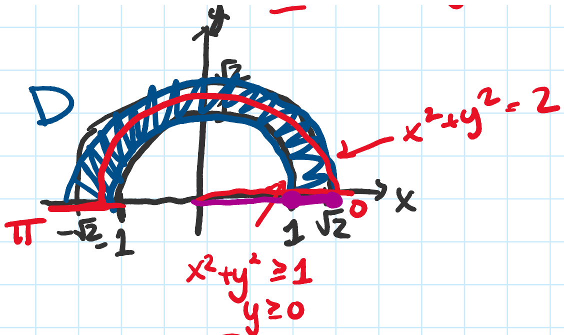
Equation of a Circle:

$x^2 + y^2 = r^2 \qquad r$ - radius

$1 \le x^2 + y^2 \le 2$



$D^*\ (r,\theta)$



D^* (r,θ)

$$\boxed{\begin{array}{c} 0\leq\theta\leq\pi \\ 1\leq r\leq\sqrt{2}\end{array}}$$

$$\int_{\theta=0}^{\theta=\pi}\int_{r=1}^{r=\sqrt{2}} e^{r^2}\cdot |r|\,dr\,d\theta$$

$$=\int_{\theta=0}^{\theta=\pi}\left(\int_{r=1}^{r=\sqrt{2}} re^{r^2}\,dr\right)d\theta$$

$$=\int_{\theta=0}^{\theta=\pi}\frac{1}{2}e^{r^2}\Big|_{r=1}^{r=\sqrt{2}}\,d\theta$$

$$=\int_{\theta=0}^{\theta=\pi}\left(\frac{1}{2}e^{(\sqrt{2})^2}-\frac{1}{2}e^{1^2}\right)d\theta$$

$$=\int_{\theta=0}^{\theta=\pi}\frac{1}{2}(e^2-e^1)\,d\theta$$

$$=\frac{1}{2}(e^2-e^1)\,\theta\Big|_{\theta=0}^{\theta=\pi}$$

$$=\boxed{\frac{1}{2}(e^2-e^1)\pi}$$

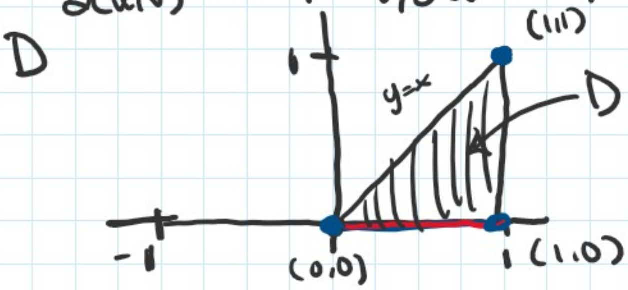
Ex Compute $\iint_D (x+y)\,dA$ where D is the region $0\leq x\leq 1$ & $0\leq y\leq x$ by using $\boxed{x=u+v}$ & $\boxed{y=u-v.}$

$$x+y=(u+v)+(u-v)=2u$$

$$x+y=(u+v)+(u-v)=2u$$

$$\frac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}\partial x/\partial u & \partial x/\partial v\\ \partial y/\partial u & \partial y/\partial v\end{vmatrix}=\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix}=-1-1=-2$$

D



D^*: $(0,0)$:

$$\begin{aligned} 0 &= u+v\\ +\;0 &= u-v\\ \hline 0 &= 2u\\ \boxed{u} &\boxed{=0}\end{aligned}$$

$$0=0+v$$

$$\boxed{v=0}$$

$$\underset{x\ \ y}{(0,0)} \longrightarrow \boxed{\underset{u\ \ v}{(0,0)}}$$

$(1,0)$:

$$\begin{aligned} 1 &= u+v\\ +\;0 &= u-v\\ \hline 1 &= 2u\\ \boxed{u} &\boxed{=1/2}\end{aligned}$$

$$0=1/2-v$$

$$\boxed{v=1/2}$$

$$\underset{x\ \ y}{(1,0)} \rightarrow \boxed{\underset{u\ \ v}{(1/2,1/2)}}$$

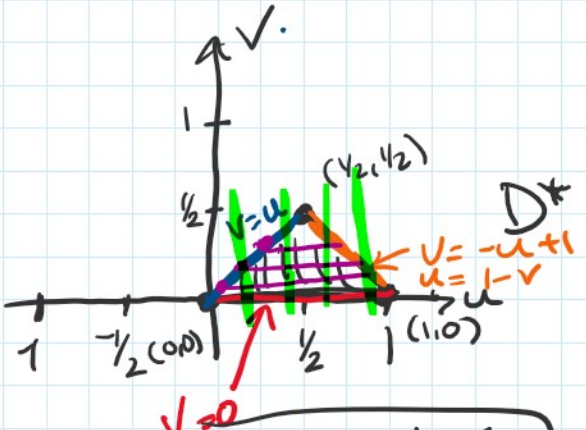
$(1,1)$:

$$\begin{aligned} 1 &= u+v\\ +\;1 &= u-v\\ \hline 2 &= 2u\\ \boxed{u} &\boxed{=1}\end{aligned}$$

$$1=1+v$$

$$\boxed{v=0}$$

$$\underset{x\ \ y}{(1,1)} \longrightarrow \boxed{\underset{u\ \ v}{(1,0)}}$$



$$\boxed{\begin{aligned} v \le\ &u \le 1-v\\ 0 \le\ &v \le 1/2\end{aligned}}$$

$y=mx+b$ $\qquad m=\dfrac{y_1-y_0}{\ldots}$ (x_0,y_0) & (x_1,y_1) are points

$y = mx+b$ $\quad m = \dfrac{y_1 - y_0}{x_1 - x_0}$ (x_0, y_0) & (x_1, y_1) are points on the line

$\rightarrow y - y_0 = m(x - x_0)$

$(0,0)$ & $(1/2, 1/2)$ $\quad m = \dfrac{1/2 - 0}{1/2 - 0} = \dfrac{1/2}{1/2} = 1$

$y - 0 = 1(x - 0)$

$y = x$

$(1/2, 1/2)$ & $(1,0)$

$m = \dfrac{0 - 1/2}{1 - 1/2} = \dfrac{-1/2}{1/2} = -1$

$(y - 1/2) = -1(x - 1/2)$

$y - 1/2 = -x + 1/2$

$y = -x + 1$

$v = -u + 1$

$$\int_{v=0}^{v=1/2} \int_{u=v}^{u=1-v} 2u \cdot |-2| \, du \, dv$$

$$= \int_{v=0}^{v=1/2} \int_{u=v}^{u=1-v} 4u \, du \, dv$$

$$= \int_{v=0}^{v=1/2} 2u^2 \Big|_{u=v}^{u=1-v} dv$$

$$= \int_{v=0}^{v=1/2} \left(2(1-v)^2 - 2v^2\right) dv$$

$$= \int_{v=0}^{v=1/2} \left(2(1 - 2v + v^2) - 2v^2\right) dv$$

$$= \int_{v=0}^{v=1/2} \left(2 - 4v + \cancel{2v^2} - \cancel{2v^2}\right) dv$$

$$\int^{v=1/2}$$

$$= \int_{v=0}^{v=1/2} (2-4v)\,dv$$

$$= (2v - 2v^2)\Big|_{v=0}^{v=1/2}$$

$$= 2\left(\tfrac{1}{2}\right) - 2\left(\tfrac{1}{2}\right)^2$$

$$= 1 - 2\cdot 1/4$$

$$= 1 - 1/2$$

$$= 1/2$$

Ex Compute the integral:

$$\int_{-1}^{1} \int_{0}^{\boxed{\sqrt{1-x^2}}} \arctan\left(\frac{y}{x}\right) dy\, dx$$

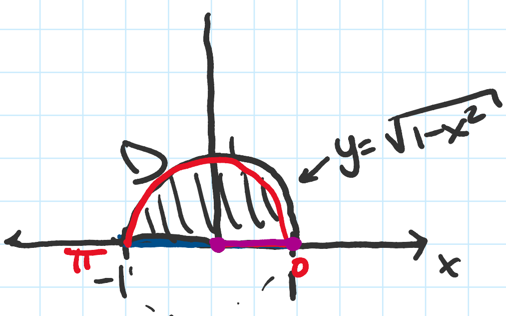
$$\boxed{x = r\cos\theta \qquad y = r\sin\theta}$$

$$\arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}\right) = \arctan\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$= \arctan(\tan(\theta)) = \theta$$

inverses

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \partial x/\partial r & \partial x/\partial\theta \\ \partial y/\partial r & \partial y/\partial\theta \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = \boxed{r}$$



$$y = \sqrt{1-x^2}$$
$$y^2 = 1 - x^2$$
$$y^2 + x^2 = 1$$ ← Equation of a circle with radius $\sqrt{1} = 1$

$$\boxed{\begin{array}{c} 0 \le \theta \le \pi \\ 0 \le r \le 1 \end{array}}$$

$$\int_{\theta=0}^{\theta=\pi}\int_{r=0}^{r=1} \theta\, |r| \, dr\, d\theta$$

$$= \int_{\theta=0}^{\theta=\pi}\int_{r=0}^{r=1} \theta r \, dr\, d\theta$$

$$= \int_{\theta=0}^{\theta=\pi} \frac{\theta r^2}{2}\Big|_{r=0}^{r=1} \, d\theta$$

$$= \int_{\theta=0}^{\theta=\pi} \frac{\theta}{2} \, d\theta$$

$$= \frac{\theta^2}{4}\Big|_{\theta=0}^{\theta=\pi}$$

$$= \boxed{\frac{\pi^2}{4}}$$